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研 究 終 了 報 告 書

「構造化制約付き最適化問題の効率的な解法の開発と機械学習への応用」

研究期間： 2021 年 10 月～2024 年 3 月
研 究 者： 劉 田香

1． 研究のねらい

The goal of this project is to solve some structured constrained optimization problems which have wide and important applications in contemporary applications. These constraints are typically hard in the following two senses: the constraint set is not simple thus one cannot expect to apply classical projection-based methods to the problems; a solution with high accuracy of feasibility of the constraints is typically required, but traditional penalty methods for handling the constraints usually lead to high computational cost and numerical instability for large-scale problems. One specific example of such constraints is the ordered constraint appearing in time-lagged regression, which has potential applications in financial time series and dynamic patient outcomes. Another example is the Hankel structured low-rank constraint (Hankel structure composite with low-rank structure), which frequently appears in the problems arising from the fields of system identification, system theory and signal processing.

In this project, we focus on two kinds of structured constrained problems. The first one is regression problems with the ordered constraint. The second one is the problems with composite-structured constraints which includes the Hankel structured low-rank constraint. We aim for the development of new efficient algorithms for these problems as well as the foundation of corresponding theoretical analysis.

2． 研究成果

（1）概要

Our research is on the following two topics.

Research topic A「Regression problems with the ordered constraint」

In this research topic, we considered the regression problems with two kinds of regularization functions for inducing the sparsity in solution: commonly used continuous regularization functions and regularization function l0 norm which is not continuous. The former has better properties such as continuity while the latter characterizes sparsity directly.

Our research on the former one achieved great success: we utilized the structure of the ordered constraint and proposed an efficient algorithm; we analyzed theoretical properties of our algorithm and also illustrated its efficiency via numerical experiments for predicting meteorological data; see [1] in Section 5 (主な研究成果リスト).

Our research on the latter one does not generally output expected numerical performance, but we improved the performance of existing methods in some scenarios. We did not submit the research result to any journal, but it formed a piece of a master student’s thesis in our lab.

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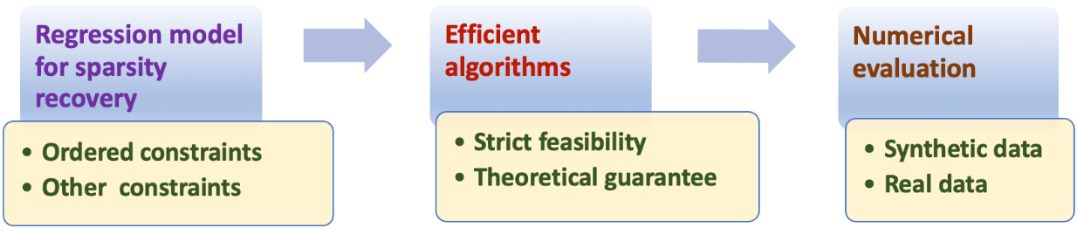
Research topic B「Optimization problems with composite-structured constraints」

In this research topic, we considered the class of constraints having a composite structure of a linear structure and another simple structure. Our research was divided into two parts. First, for a sparsity recovery model with such a constraint., we developed an efficient algorithm for the model. We did not submit the corresponding results to any journal but let it be a piece of a master student's thesis in our lab. In the second part, we focused on the feasibility issue and conducted convergence theory which explicitly connected the convergence rates with the structures on the constraint set; see [2] in Section 5 (主な研究成果リスト).

(2)詳細

研究テーマ A「Regression problems with the ordered constraint」

We introduce our research results in [1]. The research basically followed our plan as below.

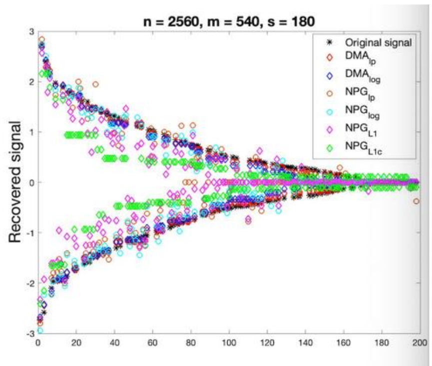


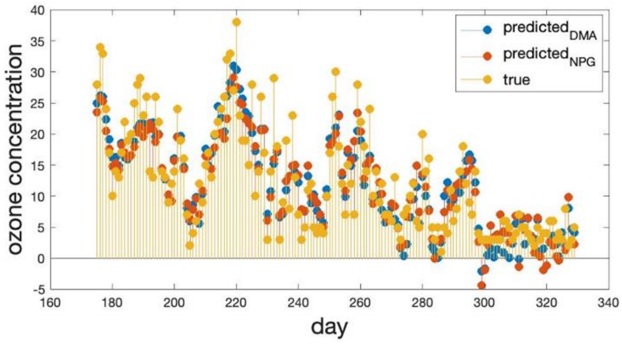
First, we proposed a general sparse regression model with a structured constraint which includes but is not limited to the ordered constraint. The model can be widely used in many time-lagged applications, such as recovering a decaying NMR signal and predicting meteorological data. Since the model involves two non-smooth parts (the regularization function and the constraint), there was no existing efficient algorithm with theoretical guarantee.

In [1], we proposed an efficient doubly majorized algorithm (DMA) for the model. The efficiency of DMA benefits from two aspects: in the inner loop we did not pursue an exact solution but only required an inexact solution; by utilizing the structure of the regularization function and the constraint, each subproblem is easy to solve. The efficiency allows its applicability to large-scale problems in machine learning and statistics. Moreover, DMA generates sequences for which the strict feasibility of constraint is maintained.

We also built the convergence theory for DMA, which guarantees the robustness of the algorithm. Since DMA is an inexact algorithm, we cannot obtain a solution which satisfies traditional optimality condition. Therefore, we proposed a new concept of optimality condition to characterize the quality of a good solution, and showed that the sequences of DMA converge to such a solution.

Finally, we applied DMA to synthetic data and real data to verify its efficiency. The comparison results of DMA and existing methods for solving approximate models are as shown in the following figures. In the left figure, it shows that two versions of DMA (red and blue) outperform other methods in terms of recovery of true synthetic signals (black stars). We also used real data of ozone concentration for prediction of future data. In the right figure, it shows that our method outputs a slightly better prediction of ozone concentration than existing methods.

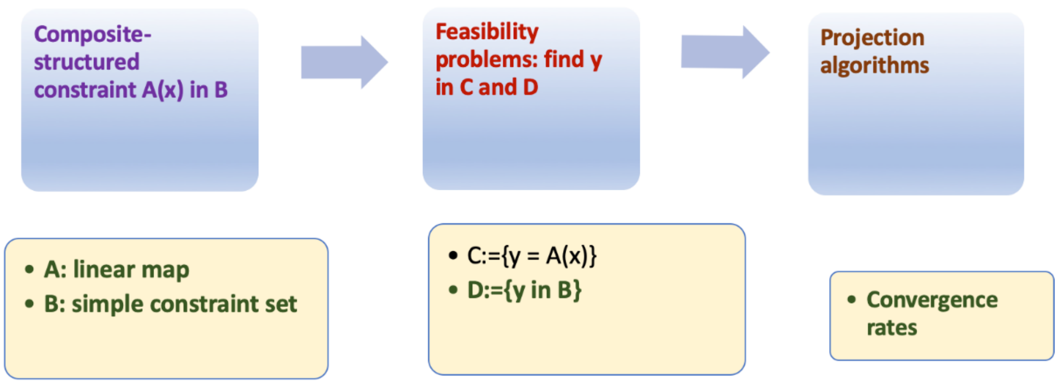




研究テーマ B「Optimization problems with composite-structured constraints」

In this research topic, optimization problems with the following class of constraints were considered: constraints having a composite structure of a linear structure and another simple convex structure. To handle such structured constraints, purely applying some penalty methods usually causes numerical issues, but it is possible to consider a hybrid method composed with a penalty method and a post-processing for refining the feasibility of solution.

Next, we introduce our research results in [2], which covers the theoretical aspects of the post-processing involving the feasibility issue of composite-structured constraints. The research content and flow are shown as below.



First, we can rewrite the feasibility problem (find x such that A(x) in B) as a new feasibility problem which is to find a common point in two simpler sets C and D. This successfully splits the difficulty of a composite structures into two simple structure. Consequently, many classical projection algorithms can be applied to the new feasibility problem.

However, it was unknown how fast these projection algorithms are. In [2], we answered this question. Specifically, we explicitly connected the convergence rates of many projection algorithms with the geometry of the sets C and D, which is further related to the structure of the original composite constraint set {x: A(x) in B}. Our theoretical analysis, when applied to many conic optimization problems, lead to some new results. For example, the convergence

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rates of more projection algorithms are connected to a quantity that characterizes the difficulty of the problem. We also tested our theoretical results in [2] on the feasibility problem on some specific sets and verified the tightness of our analyzed convergence rates of several projection algorithms.

3． 今後の展開

The future research based on the results from this project basically follows the following three directions.

1. For the successfully designed algorithm in this project, I plan to study more of its potential applications in society. I would also investigate whether some additional restrictions (for example, computational time) exists in real world. If so, its new variants of methods such as acceleration methods based on the algorithm will be explored. (about 1.5 years)
2. For the theoretical progress in this project, I plan to further extend our results to a more general setting (for example, nonconvex setting) in the future 3–5 years. Actually, an ongoing work is along this direction.
3. Finally, for the designed algorithms in this project which did not result in expected performance, I plan to further modify them to improve the efficiency. (about 1 year)

4． 自己評価

Overall, nearly 70% of the project's original research goals were achieved. Although some of the designed algorithms did not have the expected numerical performance, we are still exploring how to further improve them. Currently, the main achievements include a successful design of an algorithm and a foundation of theoretical analysis. The algorithm itself can be widely used in various application problems in various fields such as economy and medicine science. In addition, the design idea of this algorithm is also inspiring for future algorithm design for problems with similar structures. As for the achievement in theoretical aspect, I think this is a relatively big progress in the development of optimization convergence theory. In particular, we have introduced a totally new theoretical analysis tool for the first time, and we believe that the application of this tool will receive tremendous development in the future.

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5. 主な研究成果リスト

（1）代表的な論文（原著論文）発表

研究期間累積件数： 2件

[1]. Tianxiang Liu, Ting Kei Pong and Akiko Takeda. Doubly majorized algorithm for sparsity-inducing optimization problems with regularizer-compatible constraints. Computational Optimization and Applications. 2023, 86, 521-553.
In this paper, we consider a sparsity-inducing regression model with a class of structured constraints including the ordered constraint. The presence of both the sparsity-inducing regularization function and the constraint set poses challenges on the design of efficient algorithms. By exploiting the properties in the sparsity-inducing regularization function and the structured constraint, we propose a new efficient algorithm DMA with theoretical guarantee. We also illustrate numerically the performance of DMA on synthetic data for signal recovery and on real data for the prediction of ozone concentration.
[2]. Tianxiang Liu and Bruno F. Lourenço. Convergence analysis under consistent error bounds. Foundations of Computational Mathematics. 2024, 24, 429-479.
In this paper, we conduct a systemic convergence rate analysis for convex feasibility problem. Our main result is that the convergence rate of several projection algorithms for feasibility problems can be expressed explicitly in terms of the underlying consistent error bound function, which is related to the structures of the sets. We also introduce the usage of Karamata theory which allows us to reason about convergence rates while bypassing certain complicated expressions. Finally, applications to conic feasibility problems are given and we show that a number of algorithms have convergence rates depending explicitly on a quantity of the problem.

(2) 特許出願

研究期間全出願件数： 0 件

（3）その他の成果（主要な学会発表、受賞、著作物、プレスリリース等）

1. 2023 年 11 月. 発表: 第 35 回 RAMP 数理最適化シンポジウム (RAMP 2023)
2. 2023 年 9 月. 発表: 日本 OR 学会秋季研究発表会
3. 2023 年 9 月. 日本オペレーションズ・リサーチ学会の研究賞奨励賞
4. 2023 年 8 月. 発表: 10th International Congress on Industrial and Applied Mathematics (ICIAM 2023)
5. 2022 年 8 月. 発表: 最適化研究集会「数理最適化：モデル, 理論, アルゴリズム」